Frequency Quality Metrics based on Second-Order Derivative and Autocorrelation

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***Abstract*—This industry-oriented paper originates from the observation that current frequency quality metrics utilized by transmission system operators (TSOs) fail to fully capture the dynamic behavior of the grid frequency. Motivated by this gap, the paper proposes novel frequency quality metrics based on second-order dynamics and stochastic autocorrelation. Using real-world data from the Irish, Great Britain and Nordic systems and running dynamic stochastic simulations, the paper shows that the proposed metrics bring new and counterintuitive insights in terms of how good or poor the frequency quality of power grids is beyond current well-known metrics. In particular, the paper shows that a power system may show good frequency quality using standard metrics and poor frequency quality using the proposed metrics. Overall, the paper contributes to improve the understanding of frequency quality.**

***Index Terms*—Frequency quality, metrics, rate of change of frequency (RoCoF), autocorrelation.**

I. INTRODUCTION

Frequency quality is a topic of huge interest to transmission system operators (TSOs) [1]. To quantify it, TSOs utilize several different metrics. Among them, minutes outside defined ranges such as ±100 mHz or ±200 mHz, standard deviation of the frequency, σ_f, and rate of change of frequency (RoCoF) are few of the most important ones. While these metrics are extremely useful for TSOs, they do not capture important dynamics of system frequency such as its second-order dynamics and autocorrelation. The latter describe important features of stochastic processes such as temporal dependencies and memory.

In terms of power system applications, references [2]–[6] are among the very few works that utilize the second-order derivative of frequency or its autocorrelation function (ACF). In particular, while the authors in [3], [5], [6] analyze the ACF of some real-world grids, including of the All-Island power system (AIPS) of Ireland, Great Britain (GB) and Nordic, they do not discuss what additional information ACF brings compared to existing metrics aimed at quantifying frequency quality. In this context, this industry-oriented work brings the following novel contributions:

- Proposes novel frequency quality metrics based on second-order derivatives of system frequency and its ACF.
- Shows through real-world data and dynamic stochastic simulations that the proposed metrics provide useful operational insights to TSOs.

II. CONVENTIONAL METRICS

Table I presents conventional frequency quality metrics and their operational limits, if any, for the AIPS, GB and the Nordic grids.

TABLE I
KEY FREQUENCY QUALITY METRICS UTILIZED BY TSOS

System	Minutes outside ±100, ±200 mHz (mins)	σ_f (Hz)	RoCoF (Hz/s)
AIPS	15,000 (±200 mHz)	–	1
GB	15,000 (±200 mHz)	–	0.5
Nordic	15,000 (±100 mHz)	–	–

The standard deviation (σ_f) of a time series is defined as:

$$\sigma_f = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(f_t - \overline{f})^2}\,, \tag{1}$$

where f_t represents the instantaneous frequency at time t, $\overline{f}$ represents the mean frequency, and N represents the total number of frequency time series data.

RoCoF is calculated as follows:

$$\mathrm{RoCoF} = \frac{f_t - f_{t-\tau}}{\tau}\,, \tag{2}$$

where τ represents the time interval.

III. PROPOSED METRICS

1) Rate of Change of RoCoF: Power systems worldwide are experiencing faster dynamics due to the rapid growth of highly variable demand and generation technologies. While RoCoF is an extremely important metric for TSOs it does not capture all the fast and nonlinear dynamics namely the second-order ones. For this reason, we take its derivative, RoCoF':

$$\mathrm{RoCoF}' = \frac{\mathrm{RoCoF}^{\tau}_{\mathrm{t}} - \mathrm{RoCoF}^{\tau}_{\mathrm{t}-\Delta\tau}}{\Delta\tau}\,, \tag{3}$$

where, similar to τ, $\Delta\tau$ is a time window parameter for RoCoF' calculation. To quantify and get an understanding of the long-term stochastic evolution of second-order derivatives, we calculate the standard deviation of RoCoF', $\sigma_{\mathrm{RoCoF}'}$:

$$\sigma_{\mathrm{RoCoF}'} = \sqrt{\frac{1}{N-2}\sum_{t=1}^{N-2}(\mathrm{RoCoF}'_t - \overline{\mathrm{RoCoF}}')^2}\,. \tag{4}$$

2) Autocorrelation: The ACF of a stochastic process, in this case, of the frequency time series data, is the measure of correlation of the current values to the past values of the frequency. It measures the linear dependence of the frequency to the delayed version of the same frequency over progressive time delays. This aspect of frequency quality is yet to be found under existing metrics and, as shown below, contributes to an

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increased understanding by TSOs. This is consistent with the findings of recent literature, such as [4], which indicates that a high value of the autocorrelation of a stochastic process can lead to high variations of the system variables and, in some cases, to instability. It is important that TSOs take into account such results when evaluating the performance of their grids.

The ACF can be expressed as a function of time delay θ, and is written as follows:

$$R_{\kappa_f}(\theta) = \frac{\mathrm{E}[(\kappa_f(t) - \mu_{\kappa_f})(\kappa_f(t+\theta) - \mu_{\kappa_f})]}{\sigma^2_{\kappa_f}}, \qquad (5)$$

where R_{κ_f} is the ACF of the stochastic process κ_f; and μ_{κ_f} and $\sigma^2_{\kappa_f}$ are the mean and variance of κ_f, respectively.

In order to quantify frequency quality based on $R_{\kappa_f}(\theta)$ we need to fit it to relevant functions such to a sum of damped sinusoidal and decaying exponential functions and then use the parameters of these functions such as the exponential parameter α, as important frequency quality metrics. In this work, the fitted ACF ($R_{\kappa_f}(\theta)$) is approximated as:

$$R^{\text{fitted}}_{\kappa_f}(\theta) = \underbrace{u_1 e^{-\alpha_{\text{fast}}\theta}}_{\text{term}_1} + \underbrace{(1-u_1)e^{-\alpha_{\text{slow}}\theta}\cos(\omega\theta)}_{\text{term}_2}, \qquad (6)$$

where u_1 represents the weight of the initial fast decay component; α_{fast} represents the decay rate of the initial "shock" drop; α_{slow} represents the decay rate of the oscillatory component; and ω represents the angular frequency of the oscillation. Equation (6) is fitted to $R_{\kappa_f}(\theta)$ by utilizing a non-linear least squares method, included in the Python package SciPy [7].

As illustrated in the next sections, (6) appears as a good option to fit the ACF of power system frequency. Note that from a frequency quality perspective lower values of α_{fast}, α_{slow}, and ω indicate good frequency quality as it means frequency time series values are correlated and do not change fast in different timescales (α_{fast}, α_{slow}), as well as do not oscillate (ω). The limit case is when we have steady-state/constant frequency which leads to zero values for all parameters. Conversely, high values of α_{fast}, α_{slow}, and ω indicate poor frequency quality.

IV. REAL-WORLD DATA

To calculate the proposed frequency quality metrics, we utilize AIPS, GB, and Nordic frequency time-series data from September 2025 and, for comparison, some data from December 2025, with 1 s resolution. Note that it has been shown in the literature that one month data and analysis is sufficient to generalize frequency quality behavior [8]. As available data have 1 s resolution, we assume static time intervals to calculate RoCoF and RoCoF′, that is, τ = 1 s and $\Delta\tau$ = 1 s in (2) and (3), respectively.

A. September 2025

Figure 1 depicts relevant frequency traces of the three power systems using, for illustration purposes, the first 10,000 data points of 1st September 2025 (random window selection). All three systems show stochastic behavior of system frequency with relevant jumps as well. In particular, the GB grid, despite being much bigger than AIPS and slightly lower than Nordic, demonstrates more volatile frequency around 50 Hz.

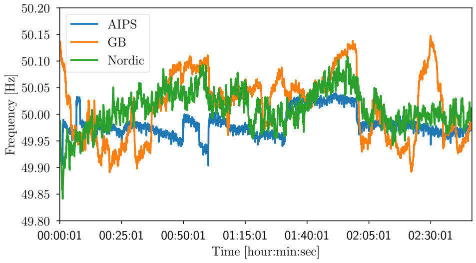


Fig. 1. Frequency variations for a selected random time window.

Table II shows that the Nordic grid exhibits superior frequency quality when referring to conventional metrics (σ_f = 0.032 Hz). Similar conclusion may be reached if we calculate the minutes outside defined ranges (see Table I) where the AIPS, GB and the Nordic demonstrate 0, 8.91, and 1.55 minutes, respectively. This is due to the Nordic TSOs implementing automatic generation control (AGC) while that is not the case in the AIPS and GB. However, if we calculate the standard deviation of the RoCoF (σ_{RoCoF}) and $\sigma_{\mathrm{RoCoF'}}$ the situation changes. That is, the performance of Nordic is low with respect to the proposed metric ($\sigma_{\mathrm{RoCoF'}}$ = 0.0040 Hz/s^2). This is a counterintuitive result given the Nordic grid has a bigger size and lower σ_f than AIPS and GB. Based on a time series of 10,000 data points, the Nordic grid shows the highest $\sigma_{\mathrm{RoCoF'}}$ (0.0036 Hz/s^2) compared to the AIPS ($\sigma_{\mathrm{RoCoF'}}$ = 0.0017 Hz/s^2) and GB ($\sigma_{\mathrm{RoCoF'}}$ = 0.0027 Hz/s^2).

TABLE II
FREQUENCY QUALITY METRICS FOR SEPTEMBER 2025

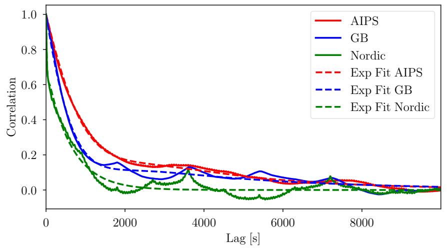
System	σ_f (Hz)	σ_{RoCoF} (Hz/s)	$\sigma_{\mathrm{RoCoF'}}$ (Hz/s^2)	u_1	α_{fast}	α_{slow}	ω
AIPS	0.041	0.0022	0.0020	0.3931	0.0003	0.0013	0.0012
GB	0.074	0.0024	0.0029	0.2249	0.0003	0.0017	0.0013
Nordic	0.032	0.0031	0.0040	0.7016	0.0016	0.0422	0.0419

Figure 2 supports this conclusion by showing that the Nordic grid has a very fast decaying ACF in the initial time lags. Relevant parameters of the fitting function (6) in Table II namely α_{fast} and α_{slow} confirm these results by showing higher values compared to the GB and AIPS. This result is consistent as, despite all three power systems have significant share of variable wind and solar generation, the AIPS has a lower level of load volatility relative to its size compared to what is the case in GB and the Nordics. As a matter of fact, it has been shown in the literature that the rate of change of demand is one of the main causes of frequency events in GB [9]. Similarly, load fluctuations with a wide frequency range (i.e., approximated to white noise) are one of the main sources of ultra low-frequency oscillations in the Nordic grid [10]. These oscillations have a negative impact on frequency quality.

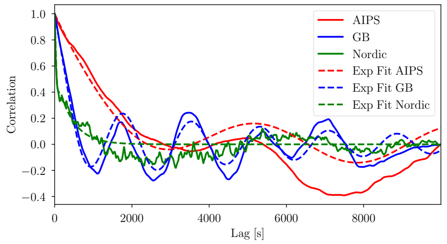
Figure 2 also reveals correlation peaks every 15/30 minutes for GB and the Nordic synchronous areas which indicate periodicity in the data. These peaks are due to market-driven imbalances that are more pronounced in self-dispatch systems (Nordic and GB) rather than central-dispatch ones (AIPS).

B. December 2025

To complement the analysis of the previous section, we select another random period namely the first 10,000 data



(a) September 2025



(b) December 2025

Fig. 2. ACFs of the frequency and their fitted versions for real-world data.

points from 1st December 2025 and apply the proposed metrics. Figure 2 and Table III confirm the previous results and show that the Nordic grid exhibits a rapid decaying ACF and higher values of α_{fast} and α_{slow} compared to the AIPS and GB power systems. This is despite it showing lower σ_f (0.023 Hz) compared to the other two systems. All three systems demonstrate 0 minutes outside defined ranges.

TABLE III
FREQUENCY QUALITY METRICS FOR DECEMBER 2025

System	σ_f (Hz)	σ_{RoCoF} (Hz/s)	$\sigma_{\mathrm{RoCoF}'}$ $(\mathrm{Hz}/\mathrm{s}^2)$	u_1	α_{fast}	α_{slow}	ω
AIPS	0.041	0.0015	0.0014	0.857	0.0008	0.0000	0.0012
GB	0.059	0.0027	0.0041	0.7100	0.0024	0.0001	0.0035
Nordic	0.023	0.0018	0.0019	0.5798	0.0025	0.0207	0.0297

Compared to Table II, Table III shows that σ_{RoCoF} and $\sigma_{\mathrm{RoCoF}'}$ for Nordic are lower (0.0018 and 0.0019 Hz/s^2, respectively) than σ_{RoCoF} and $\sigma_{\mathrm{RoCoF}'}$ for GB (0.0027 and 0.0041 Hz/s^2, respectively). This indicates higher rate of change of demand in GB. Moreover, the GB system shows a higher oscillating behavior indicating strong periodicity in the time series data. Both metrics complement each other and provide additional information to TSOs beyond existing conventional metrics.

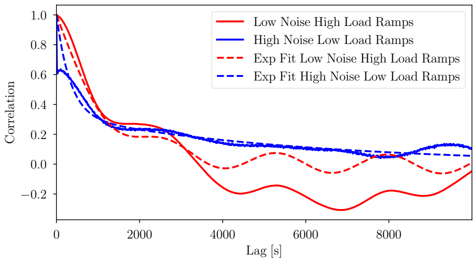


Fig. 3. ACFs of the frequency for simulations and their fitted versions.

TABLE IV
FREQUENCY QUALITY METRICS FOR THE STOCHASTIC SIMULATIONS

System	σ_f (Hz)	σ_{RoCoF} (Hz/s)	$\sigma_{\mathrm{RoCoF}'}$ $(\mathrm{Hz}/\mathrm{s}^2)$	u_1	α_{fast}	α_{slow}	ω
Low noise & high ramps	0.032	0.00093	0.0010	0.9374	0.0008	0.0000	0.0024
High noise & low ramps	0.015	0.0038	0.0041	0.3277	0.0002	0.0030	0.0000

V. CASE STUDY

To validate the real-world data, we employ the IEEE 9-bus system and run 24 h dynamic stochastic simulations using Dome [11]. Two scenarios are simulated namely one with high noise and relatively low load ramps (e.g., Nordic) and one with low noise and high load ramps (e.g., AIPS). Primary frequency control is the same in both cases and no AGC is assumed for simplicity. Figure 3 and Table IV confirm the real-world results (e.g., Section IV-B). That is, while the case with low noise and high load ramps leads to higher σ_f (0.032 Hz) it also shows lower $\sigma_{\mathrm{RoCoF}'}$ (0.0010 Hz/s^2) and α_{slow} (0.0000). Conversely, the case with high noise and low load ramps demonstrates lower σ_f (0.015 Hz) but higher $\sigma_{\mathrm{RoCoF}'}$ (0.0041 Hz/s^2) and, in turn, shows a rapid decaying ACF (first 10,000 data points) and higher α_{slow} (0.0030). Finally, both scenarios demonstrate 0 minutes outside ±200 mHz range.

VI. CONCLUSIONS

This letter demonstrates that current frequency quality metrics utilized by TSOs do not fully capture key aspects of frequency such as its second-order dynamics and ACF which describe important properties of stochastic processes. The letter proposes novel metrics based on the latter and apply those to operational data from the AIPS, GB and Nordic, as well as to dynamic stochastic simulations. It is shown that the proposed metrics contribute to an increased understanding of frequency quality. For example, the letter shows that a power system can demonstrate good frequency quality when applying the existing metrics and poor frequency quality when applying the proposed metrics (e.g., low σ_f but high $\sigma_{\mathrm{RoCoF}'}$).

Future work will focus on evaluating and record over a reporting period the proposed metrics in real-world power systems with the goal of identifying the sources of poor frequency quality and designing novel controllers.

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